

Non-BPS Brane Dynamics And Dual Tensor Gauge Theory

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Abstract

The action for the long wavelength oscillations of a non-BPS $p=3$ brane embedded in $N=1$, $D=5$ superspace is determined by means of the coset method. The $D=4$ world volume Nambu-Goldstone boson of broken translation invariance and the two $D=4$ world volume Weyl spinor Goldstinos of the completely broken supersymmetry describe the excitations of the brane into the broken space and superspace directions. The resulting action is an invariant synthesis of the Akulov-Volkov and Nambu-Goto actions. The $D=4$ antisymmetric tensor gauge theory action dual to the $p=3$ brane action is determined.

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1 Introduction

Brane world scenarios in which our four dimensional ($D=4$) world is assumed to be realized on a solitonic brane embedded in a higher dimensional space have been the topic of much research activity [1]. Superstring theories, which may possibly provide the ultraviolet completion for such models, also require supersymmetry (SUSY) in higher dimensional space-time. The minimum number of supercharges in $D=5$ or $D=6$ is eight, hence one is led to consider solitons in SUSY theories with at least eight supercharges. BPS solitons preserve some fraction of the SUSY of the underlying model and so the soliton localized effective field theory also possesses it. $\frac{1}{2}$ -BPS domain walls breaking eight supersymmetries down to four were discussed in nonlinear sigma models as well as gauge theories (see [2]-[3] and references therein).⁴ $\frac{1}{2}$ -BPS vortices in $N=1$, $D=6$ (eight supersymmetries) gauge theories were also discussed (see [5] and the references therein). Since the effective field theory on BPS solitons is still supersymmetric, in the end, their SUSY must be broken if they are to realize our world. This problem can be resolved if stable non-BPS branes exist which break SUSY completely on their world volume theory.

A non-BPS domain wall in a generalized $N=1$, $D=4$ (four supersymmetries) Wess-Zumino model was discovered in which its world volume is $N=0$, $D=3$ (no supersymmetries) [6]. The long wavelength effective action for such a non-BPS brane was constructed using the method of nonlinear realizations [7]-[10] in case the $N=1$, $D=4$ super-Poincaré symmetries are broken down to $N=0$, $D=3$ Poincaré symmetries [11] and equivalently by the Green-Schwarz method [12], as generalized by Sen [13] to the non-BPS case [14]. Since the above model with a $D=3$ world volume is a toy model for the brane world picture, higher dimensional models are desired in order to realize a non-BPS $D=4$ world-volume. Some $N=1$, $D=5$ (eight supersymmetries) short distance models admitting non-BPS walls have been found recently [15] and the second reference in [3]. In the former [15], a periodic configuration of BPS and anti-BPS walls was considered in a $N=1$, $D=5$ $U(1)$ SUSY gauge theory with two

⁴Other types of solitons in these models were discussed in [4].

hypermultiplets of equal charge. There it was shown that the configuration was stable for small fluctuations and meta-stable for large fluctuations. Of further interest is the second model in the second reference in [3] in which the existence of non-BPS walls was shown, although no explicit solution has been constructed yet. Briefly, this model consists of a D=5, N=1 SUSY $U(N_C)$ gauge theory with N_F hypermultiplets belonging to the fundamental representation. If the Fayet-Iliopoulos term is added, the model contains $N_F!/(N_C!(N_F - N_C)!)$ discrete degenerate vacua [16]. The general solution for $\frac{1}{2}$ -BPS domain walls connecting these vacua was constructed [3]. However, there exists one pair of vacua which are connected by a non-BPS domain wall in the case of $N_F = 4$ and $N_C = 2$. There exist more non-BPS walls connecting different sets of vacua for larger N_F and/or N_C .

The purpose of this paper is to construct the low energy thin domain wall effective action for the non-BPS case by means of the coset method. In particular the nonlinear realization describing the breakdown of the N=1, D=5 super-Poincaré group to the (N=0) D=4 Poincaré group is given. The appendix contains the N=1, D=5 SUSY algebra expressed in terms of the unbroken D=4 Lorentz group decomposition of the charges as a centrally extended N=2, D=4 SUSY algebra. The low energy D=4 world volume fields consist of the Nambu-Goldstone boson scalar field ϕ corresponding to the broken D=5 space translation invariance and the two D=4 Weyl spinor Goldstino fields θ_α and λ_α , $\alpha = 1, 2$, of the broken N=1, D=5 supersymmetry.

In section 2, the coset method is used in order to construct the effective action for the domain wall oscillations in the thin wall limit. This p=3 brane action is an invariant synthesis of the Akulov-Volkov and Nambu-Goto actions with ϕ describing the space oscillations of the brane and θ and λ describing the oscillations of the brane into the Grassmann directions of N=1, D=5 superspace. The static gauge action has the form of the determinant of the induced metric vierbein e_μ^a

$$\Gamma = -\sigma \int d^4x \det e = -\sigma \int d^4x \det \hat{e} \det N, \quad (1.1)$$

where σ is the brane tension, \hat{e}_μ^a is the Akulov-Volkov vierbein

$$\hat{e}_\mu^a = \delta_\mu^a + i(\theta \overset{\leftrightarrow}{\partial}_\mu \sigma^a \bar{\theta} + \lambda \overset{\leftrightarrow}{\partial}_\mu \sigma^a \bar{\lambda}) \quad (1.2)$$

and N_a^b is the Nambu-Goto vierbein. After application of the “inverse Higgs mechanism” [17], $N_a^b = \delta_a^b + \frac{\hat{\nabla}_a \phi \hat{\nabla}^b \phi}{(\hat{\nabla} \phi)^2} \left(\sqrt{1 - (\hat{\nabla} \phi)^2} - 1 \right)$, where the Nambu-Goldstone boson covariant derivative, $\hat{\nabla}_a \phi$, is defined as $\hat{\nabla}_a \phi = \hat{\mathcal{D}}_a(\phi + i(\theta \lambda - \bar{\theta} \bar{\lambda}))$, with the Akulov-Volkov partial covariant derivative, $\hat{\mathcal{D}}_a$, given by $\hat{\mathcal{D}}_a = \hat{e}_a^{-1\mu} \partial_\mu$. The determinant of the Nambu-Goto vierbein yields

$$\det N = \sqrt{1 - [\hat{\mathcal{D}}_a(\phi + i(\theta \lambda - \bar{\theta} \bar{\lambda}))]^2}. \quad (1.3)$$

In D=4 an antisymmetric tensor gauge theory can be used to equivalently describe the Nambu-Goldstone bosons of broken internal symmetries [18]. Bagger and Galperin showed that a tensor field may be regarded as a Nambu-Goldstone mode for broken translational symmetry in the case of partially broken supersymmetry on a BPS soliton [19]. In section 3 the tensor gauge theory action dual to the Akulov-Volkov-Nambu-Goto action, equation (1.1), is constructed. This construction demonstrates the similar role played by the tensor gauge field in the non-BPS domain wall case as in the internal symmetry [18] and BPS domain wall [19] cases. It is a supersymmetric generalization of the tensor gauge theory action dual to the p=3 bosonic brane action. The bosonic brane embedded in D=5 space-time has the Nambu-Goto action

$$\Gamma_{\text{NG}} = -\sigma \int d^4x \sqrt{1 - \partial_\mu \phi \partial^\mu \phi}. \quad (1.4)$$

The dual tensor gauge theory action can be found by introducing the Lagrange multiplier [20] field strength F_μ so that $V_\mu = \partial_\mu \phi$

$$\Gamma_{\text{NG}} = -\sigma \int d^4x \left[\sqrt{1 - V_\mu V^\mu} + F_\mu (V^\mu - \partial^\mu \phi) \right]. \quad (1.5)$$

The Nambu-Goldstone boson field equation implies that $\partial_\mu F^\mu = 0$ with tensor gauge field solution $F_\mu = \epsilon_{\mu\nu\rho\sigma} \partial^\nu B^{\rho\sigma}$. The vector field V_μ equation of motion is algebraic and so can be eliminated to yield the dual tensor gauge theory action

$$\Gamma_{\text{NG}} = -\sigma \int d^4x \sqrt{1 + F^\mu F_\mu}. \quad (1.6)$$

The coset method action for the N=1, D=5 super-Poincaré symmetries spontaneously broken to D=4 Poincaré symmetries includes an auxiliary vector field for the broken

D=5 Lorentz transformations. Eliminating the Nambu-Goldstone scalar boson ϕ instead of the auxiliary vector field by means of its Euler-Lagrange equation (see Ivanov, et al. in [9]) leads directly, as equivalently the Lagrange multiplier method does, to the dual tensor gauge theory action

$$\Gamma = -\sigma \int d^4x \sqrt{-\det(\hat{g}_{\mu\nu} + F_\mu F_\nu)}, \quad (1.7)$$

where $\hat{g}_{\mu\nu} = \hat{e}_\mu^a \eta_{ab} \hat{e}_\nu^b$ is the induced Akulov-Volkov metric.

2 The Coset Method And Brane Dynamics

The action for a non-BPS p=3 brane embedded in N=1, D=5 superspace can be constructed by means of the coset method for the case of the breakdown of the N=1, D=5 super-Poincaré group, denoted G , to the unbroken D=4 Poincaré and R symmetry groups, denoted $H = ISO(1, 3) \times R$. The technique begins with the coset element $\Omega \in G/SO(1, 3) \times R$

$$\Omega = e^{ix^\mu P_\mu} e^{i\phi(x)Z} e^{i[\theta^\alpha(x)Q_\alpha + \bar{\theta}_{\dot{\alpha}}(x)\bar{Q}^{\dot{\alpha}} + \lambda^\alpha(x)S_\alpha + \bar{\lambda}_{\dot{\alpha}}(x)\bar{S}^{\dot{\alpha}}]} e^{iv^\mu(x)K_\mu}, \quad (2.1)$$

where the x^μ denote the D=4 space-time coordinates parameterizing the world volume of the 3-brane in the static gauge, while the Nambu-Goldstone fields, denoted by $\phi(x)$, $\theta_\alpha(x)$, $\bar{\theta}_{\dot{\alpha}}(x)$, $\lambda_\alpha(x)$, $\bar{\lambda}_{\dot{\alpha}}(x)$ and $v^\mu(x)$, describe the co-volume target space excitations of the brane. Taken together, they act as coordinates of the coset manifold G/H . Multiplication of the coset elements Ω by group elements $g \in G$ from the left results in transformations of the space-time coordinates and the Nambu-Goldstone fields according to the general structure

$$g\Omega = \Omega' h. \quad (2.2)$$

The transformed coset element yields the world volume coordinate transformations and the total variations of the fields

$$\Omega' = e^{ix'^\mu P_\mu} e^{i\phi'(x')Z} e^{i[\theta'^\alpha(x')Q_\alpha + \bar{\theta}'_{\dot{\alpha}}(x')\bar{Q}^{\dot{\alpha}} + \lambda'^\alpha(x')S_\alpha + \bar{\lambda}'_{\dot{\alpha}}(x')\bar{S}^{\dot{\alpha}}]} e^{iv'^\mu(x')K_\mu}, \quad (2.3)$$

while h is a field dependent element of $SO(1, 3) \times R$.

According to the coset construction method, the vierbein, the covariant derivatives of the Nambu-Goldstone fields and the spin connection can be obtained from the Maurer-Cartan one-form. The Maurer-Cartan one-form can be determined by use of the Feynman formula for the variation of an exponential operator along with the Baker-Campbell-Hausdorff formula. So doing, the Maurer-Cartan one-form is secured

$$\begin{aligned}\Omega^{-1}d\Omega = & i \left[\omega^a P_a + \omega_Z Z + \omega_Q^\alpha Q_\alpha + \bar{\omega}_{Q\dot{\alpha}} \bar{Q}^{\dot{\alpha}} + \omega_{S\alpha} S_\alpha + \bar{\omega}_{S\dot{\alpha}} \bar{S}^{\dot{\alpha}} \right. \\ & \left. + \omega_K^a K_a + \omega_M^{\mu\nu} M_{\mu\nu} + \omega_R R \right],\end{aligned}\quad (2.4)$$

where the individual world volume one-forms are found to be

$$\begin{aligned}\omega^a &= \left(dx^b + i[\theta\sigma^b d\bar{\theta} - d\theta\sigma^b \bar{\theta} + \lambda\sigma^b d\bar{\lambda} - d\lambda\sigma^b \bar{\lambda}] \right) \times \\ &\quad \times \left(\delta_b^a + (\cosh 2\sqrt{v^2} - 1) \frac{v_b v^a}{v^2} \right) - d \left(\phi + i[\theta\lambda - \bar{\theta}\bar{\lambda}] \right) \frac{\sinh 2\sqrt{v^2}}{\sqrt{v^2}} v^a \\ \omega_Z &= d \left(\phi + i[\theta\lambda - \bar{\theta}\bar{\lambda}] \right) \cosh 2\sqrt{v^2} \\ &\quad - \left(dx^a + i[\theta\sigma^a d\bar{\theta} - d\theta\sigma^a \bar{\theta} + \lambda\sigma^a d\bar{\lambda} - d\lambda\sigma^a \bar{\lambda}] \right) v_a \frac{\sinh 2\sqrt{v^2}}{\sqrt{v^2}} \\ \omega_Q^\alpha &= \cosh \sqrt{v^2} d\theta^\alpha - \frac{\sinh \sqrt{v^2}}{\sqrt{v^2}} (d\bar{\lambda}\bar{\psi})^\alpha \\ \bar{\omega}_{Q\dot{\alpha}} &= \cosh \sqrt{v^2} d\bar{\theta}_{\dot{\alpha}} + \frac{\sinh \sqrt{v^2}}{\sqrt{v^2}} (d\lambda\psi)_{\dot{\alpha}} \\ \omega_S^\alpha &= \cosh \sqrt{v^2} d\lambda^\alpha + \frac{\sinh \sqrt{v^2}}{\sqrt{v^2}} (d\bar{\theta}\bar{\psi})^\alpha \\ \bar{\omega}_{S\dot{\alpha}} &= \cosh \sqrt{v^2} d\bar{\lambda}_{\dot{\alpha}} - \frac{\sinh \sqrt{v^2}}{\sqrt{v^2}} (d\theta\psi)_{\dot{\alpha}} \\ \omega_K^a &= dv^a + \frac{iv^2}{2} \sinh 2\sqrt{v^2} dv_b [\eta^{ba} - \frac{v^b v^a}{v^2}] \\ \omega_M^{ab} &= \left(\cosh 2\sqrt{v^2} - 1 \right) \frac{(v^a dv^b - v^b dv^a)}{2v^2} \\ \omega_R &= 0.\end{aligned}\quad (2.5)$$

The two sets of coordinate basis differentials dx^μ and ω^a are related to each other through the vierbein e_μ^a

$$\omega^a = dx^\mu e_\mu^a. \quad (2.6)$$

From equation (2.5) this yields, recalling $d = dx^\mu \partial_\mu$,

$$e_\mu^a = \left(\delta_\mu^b + i[\theta\sigma^b \overset{\leftrightarrow}{\partial}_\mu \bar{\theta} + \lambda\sigma^b \overset{\leftrightarrow}{\partial}_\mu \bar{\lambda}] \right) \left(\delta_b^a + \left(\cosh 2\sqrt{v^2} - 1 \right) \frac{v_b v^a}{v^2} \right)$$

$$-\frac{\sinh 2\sqrt{v^2}}{\sqrt{v^2}}v^a\partial_\mu[\phi + i(\theta\lambda - \bar{\theta}\bar{\lambda})]. \quad (2.7)$$

Under a G -transformation the vierbein transforms with one world index and one tangent space (structure group) index as

$$e'_\mu{}^a = G_\mu^{-1\nu}e_\nu{}^b L_b{}^a. \quad (2.8)$$

Using equation (2.3), $G_\nu{}^\mu = \partial x'^\mu / \partial x^\nu$ and $L_a{}^b$ is the D=4 Lorentz transformation corresponding to h and has determinant one, $\det L = 1$. The leading term in the N=1, D=5 super-Poincaré invariant action is given by the “cosmological constant” term

$$\Gamma = -\sigma \int d^4x \det e, \quad (2.9)$$

with σ denoting the brane tension parameter. The lagrangian is the constant brane tension integrated over the hyperarea of the brane. The action is invariant due to equation (2.8) and since $d^4x' = d^4x \det G$ and $\det L = 1$.

The fully covariant vierbein, $e_\mu{}^a$, can be factorized into the product of the partially covariant Akulov-Volkov vierbein $\hat{e}_\mu{}^a$,

$$\hat{e}_\mu{}^a = \delta_\mu{}^a + i \left(\theta\sigma^a \overleftrightarrow{\partial}_\mu \bar{\theta} + \lambda\sigma^a \overleftrightarrow{\partial}_\mu \bar{\lambda} \right), \quad (2.10)$$

and the Nambu-Goto vierbein $N_b{}^a$,

$$N_b{}^a = \delta_b{}^a + (\cosh 2\sqrt{v^2} - 1) \frac{v_b v^a}{v^2} - \frac{\sinh 2\sqrt{v^2}}{\sqrt{v^2}} v^a \hat{\mathcal{D}}_b [\phi + i(\theta\lambda - \bar{\theta}\bar{\lambda})], \quad (2.11)$$

where the partial covariant Akulov-Volkov derivative is defined by $\hat{\mathcal{D}}_a = \hat{e}_a^{-1\mu} \partial_\mu$,

$$e_\mu{}^a = \hat{e}_\mu{}^b N_b{}^a. \quad (2.12)$$

Thus, the invariant action involves the product of the Akulov-Volkov determinant and the determinant of the Nambu-Goto vierbein, as in equation (1.1). The latter can be evaluated to yield

$$\det N = \cosh 2\sqrt{v^2} \left[1 - \frac{\tanh 2\sqrt{v^2}}{\sqrt{v^2}} v^a \hat{\mathcal{D}}_a (\phi + i(\theta\lambda - \bar{\theta}\bar{\lambda})) \right]. \quad (2.13)$$

The vector field v^a associated with the broken D=5 Lorentz transformations appears in the action without any derivatives. Hence, it is an auxiliary field and can be eliminated by means of its equation of motion. Equivalently, the Maurer-Cartan one-form associated with the broken translation generator Z can be G -covariantly set to zero. This leads to the elimination of v^a via the “inverse Higgs mechanism” [17]. Expressing the ω_Z one-form in terms of the partially covariant one-form $\hat{\omega}^a \equiv dx^\mu \hat{e}_\mu^a = dx^a + i(\theta \sigma^a \overset{\leftrightarrow}{d} \bar{\theta} - \lambda \sigma^a \overset{\leftrightarrow}{d} \bar{\lambda})$ (hence $d = dx^\mu \partial_\mu = \hat{\omega}^a \hat{\mathcal{D}}_a$) gives

$$\omega_Z = \hat{\omega}^a \cosh 2\sqrt{v^2} \left[\hat{\mathcal{D}}_a (\phi + i(\theta\lambda - \bar{\theta}\bar{\lambda})) - \frac{\tanh 2\sqrt{v^2}}{\sqrt{v^2}} v_a \right]. \quad (2.14)$$

Setting this to zero results in the “inverse Higgs mechanism”

$$v_a \frac{\tanh 2\sqrt{v^2}}{\sqrt{v^2}} = \hat{\mathcal{D}}_a (\phi + i(\theta\lambda - \bar{\theta}\bar{\lambda})). \quad (2.15)$$

Substituting this into the determinant of the Nambu-Goto vierbein yields the SUSY generalization of the Nambu-Goto lagrangian

$$\det N = \frac{1}{\cosh 2\sqrt{v^2}} = \sqrt{1 - (\hat{\mathcal{D}}_a [\phi + i(\theta\lambda - \bar{\theta}\bar{\lambda})])^2}. \quad (2.16)$$

Hence the complete (G -invariant) $N=1$, $D=5$ super-Poincaré invariant Akulov-Volkov-Nambu-Goto action for a non-BPS $p=3$ brane embedded in $N=1$, $D=5$ superspace is given by

$$\begin{aligned} \Gamma &= -\sigma \int d^4x \det e = -\sigma \int d^4x \det \hat{e} \det N \\ &= -\sigma \int d^4x \left\{ \det \left[\delta_\mu^a + i(\theta \overset{\leftrightarrow}{\partial}_\mu \sigma^a \bar{\theta} + \lambda \overset{\leftrightarrow}{\partial}_\mu \sigma^a \bar{\lambda}) \right] \times \right. \\ &\quad \left. \times \sqrt{1 - (\hat{\mathcal{D}}_a [\phi + i(\theta\lambda - \bar{\theta}\bar{\lambda})])^2} \right\}. \end{aligned} \quad (2.17)$$

3 The Dual Tensor Gauge Theory

Returning to action (1.1) and equation (2.13) for the Nambu-Goto determinant with all fields independent, the ϕ equation of motion yields the Bianchi identity for the dual field strength vector F^μ (see Ivanov, et al. in [9])

$$0 = \frac{\delta \Gamma}{\delta \phi} = -\partial_\mu F^\mu, \quad (3.1)$$

where

$$F^\mu = (\det \hat{e})(v^a \hat{e}_a^{-1\mu}) \frac{\sinh 2\sqrt{v^2}}{\sqrt{v^2}}. \quad (3.2)$$

Since the dual of F^μ is closed, F^μ can be (locally) expressed as

$$F^\mu = \epsilon^{\mu\nu\rho\sigma} \partial_\nu B_{\rho\sigma}, \quad (3.3)$$

where the 2-form $B_{\mu\nu}$ is the tensor gauge potential. The lagrangian can be expressed as

$$\det e = \det \hat{e} \det N = \det \hat{e} \cosh 2\sqrt{v^2} - F^\mu \partial_\mu [\phi + i(\theta\lambda - \bar{\theta}\bar{\lambda})]. \quad (3.4)$$

Exploiting the definition of F^μ further so that

$$\frac{v^a v^b}{v^2} = \frac{(F^\mu \hat{e}_\mu^a)(F^\nu \hat{e}_\nu^b)}{(F\hat{e})^2} \quad (3.5)$$

results in

$$\cosh 2\sqrt{v^2} = \sqrt{\left(1 + \frac{(F\hat{e})^2}{(\det \hat{e})^2}\right)}. \quad (3.6)$$

Integrating equation (3.4) over the world volume after having substituted equation (3.6) and integrating by parts in order to set $\partial_\mu F^\mu = 0$, the tensor gauge theory action dual to the non-BPS p=3 brane Nambu-Goto-Akulov-Volkov action is obtained

$$\Gamma = -\sigma \int d^4x \sqrt{-(\det \hat{g}) + F^\mu \hat{g}_{\mu\nu} F^\nu}, \quad (3.7)$$

where the Akulov-Volkov metric is given by $\hat{g}_{\mu\nu} = \hat{e}_\mu^a \eta_{ab} \hat{e}_\nu^b$.

From the definition of the dual field strength F^μ and its expression in terms of the tensor gauge potential, it is a world volume vector density. Hence, it is convenient to define the covariant field strength F_μ according to

$$F_\mu \equiv \frac{1}{\det \hat{e}} \hat{g}_{\mu\nu} F^\nu. \quad (3.8)$$

The dual tensor gauge theory action can then be written as

$$\Gamma = -\sigma \int d^4x \sqrt{-\det(\hat{g}_{\mu\nu} + F_\mu F_\nu)}. \quad (3.9)$$

4 Discussion

In this paper the effective action describing the dynamics of the Nambu-Goldstone degrees of freedom localized on a non-BPS $p=3$ brane embedded in $N=1$, $D=5$ super-space was determined by means of the coset method. It was shown that the Nambu-Goldstone field associated with the broken translational symmetry can be described equivalently in terms of either a scalar field or an anti-symmetric tensor gauge potential. Besides the Nambu-Goldstone fields associated with the broken space-time symmetries, in principle other massless degrees of freedom may appear in the low energy effective action, depending on the details of the underlying model. For example, additional massless scalar fields occur in configurations of multiple non-interacting parallel domain walls, where moduli correspond to distances between pairs of walls. These non-Nambu-Goldstone massless modes likely appear as “matter” fields coupled to the Nambu-Goldstone modes in terms of the coset construction of the effective low energy action. In addition, if in a particular underlying model some internal global symmetries are broken by a domain wall configuration, corresponding Nambu-Goldstone modes appear in the effective action. In fact, this latter situation is often realized by solitons in non-Abelian gauge theories (see for instance [5, 21]). In this case, each such Nambu-Goldstone boson corresponds to a broken non-Abelian global internal symmetry. The dual action, therefore, is expected to contain a non-Abelian tensor (as was discussed by Freedman and Townsend in [18]), in addition to the Abelian tensor associated with the broken translational symmetry. Other directions in which the coset construction can be extended is to include more than the minimal number of supersymmetry generators or to allow for a higher number of soliton codimensions. Such generalizations would be of importance in order to construct the low energy action of massless degrees of freedom in many interesting models. For instance, a non-BPS vortex with codimension two was recently found in a theory with eight supercharges [22]. Since the low energy effective action of the model considered in this model contains Nambu-Goldstone bosons associated with internal global non-Abelian symmetries broken by the vortex, its dual action is anticipated to contain a non-Abelian tensor in addition to Abelian tensors, as discussed before.

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Appendix: N=1, D=5 Super-Poincaré Algebra

The D=5 Poincaré symmetry generators consist of the energy-momentum operator P^M and the angular momentum operator M^{MN} , with $M, N = 0, 1, 2, 3, 4$, which obey the algebra (the D=5 metric is taken to be $\eta^{MN} = (+, -, -, -, -)$)

$$\begin{aligned} [M^{MN}, M^{RS}] &= -i(\eta^{MR}M^{NS} - \eta^{MS}M^{NR} + \eta^{NR}M^{MR} - \eta^{NR}M^{MS}) \\ [M^{MN}, P^L] &= i(P^M\eta^{NL} - P^N\eta^{ML}) \quad [P^M, P^N] = 0. \end{aligned} \quad (\text{A.1})$$

The N=1, D=5 super-Poincaré algebra has in addition the 4 component complex (Dirac) supersymmetry charges \mathcal{Q}_a and $\bar{\mathcal{Q}}_a = \mathcal{Q}_b^\dagger \gamma_{ba}^0$ and the R -symmetry automorphism generator, R . The non-vanishing commutators are

$$\begin{aligned} \{\mathcal{Q}_a, \bar{\mathcal{Q}}_b\} &= +2\gamma_{ab}^M P_M & [M^{MN}, \mathcal{Q}_a] &= -\frac{1}{2}\gamma_{ab}^{MN} \mathcal{Q}_b \\ [R, \mathcal{Q}_a] &= +\mathcal{Q}_a & [R, \bar{\mathcal{Q}}_a] &= -\bar{\mathcal{Q}}_a. \end{aligned} \quad (\text{A.2})$$

The non-BPS domain wall breaks the N=1, D=5 super-Poincaré symmetry group G to the D=4 Poincaré symmetry and R symmetry groups denoted $H = ISO(1, 3) \times R$. The N=1, D=5 super-Poincaré charges can be written in terms of their unbroken SO(1,3) Lorentz group content. The unbroken symmetry group H is generated by the charges P^μ , with $\mu = 0, 1, 2, 3$, corresponding to translations in D=4 space-time of the world volume, $M^{\mu\nu}$, with $\mu, \nu = 0, 1, 2, 3$, corresponding to D=4 world volume Lorentz transformations and R corresponding to chiral R symmetry transformations. The remaining charges generating elements of G/H are the broken N=1, D=5 super-Poincaré charges. $Z = P_4$ generates translations in the broken fifth dimension and acts as the central charge in the equivalent extended N=2, D=4 SUSY algebra, $K^\mu = 2M^{4\mu}$ generates the broken D=5 Lorentz transformations. The 8 broken N=1, D=5 (N=2, D=4) supersymmetry generators are complex 4 component (Dirac) spinors,

\mathcal{Q}_a and $\bar{\mathcal{Q}}_a$. The Dirac spinors can be written in the Weyl representation in terms of the 2 component $N=2$, $D=4$ complex Weyl spinor charges, Q_α , $\bar{Q}_{\dot{\alpha}}$, S_α and $\bar{S}_{\dot{\alpha}}$, as

$$\mathcal{Q}_a = \begin{pmatrix} Q_\alpha \\ i\bar{S}^{\dot{\alpha}} \end{pmatrix} \quad \bar{\mathcal{Q}}_a = \begin{pmatrix} -iS^\alpha & \bar{Q}_{\dot{\alpha}} \end{pmatrix} \quad (A.3)$$

(The $D=5$ Dirac matrices are given in terms of the $D=4$ Dirac matrices, in the Weyl representation, $\gamma^M = (\gamma^\mu, i\gamma_5)$.)

In terms of these operators the $N=1$, $D=5$ super-Poincaré algebra of equations (A.1) and (A.2) becomes the centrally extended $N=2$, $D=4$ SUSY algebra given by

$$\begin{aligned} [M^{\mu\nu}, M^{\rho\sigma}] &= -i(\eta^{\mu\rho}M^{\nu\sigma} - \eta^{\mu\sigma}M^{\nu\rho} + \eta^{\nu\sigma}M^\mu\rho - \eta^{\nu\rho}M^\mu\nu) \\ [M^{\mu\nu}, P^\lambda] &= i(P^\mu\eta^{\nu\lambda} - P^\nu\eta^{\mu\lambda}) \quad [M^{\mu\nu}, K^\lambda] = i(K^\mu\eta^{\nu\lambda} - K^\nu\eta^{\mu\lambda}) \\ [Z, K^\mu] &= 2iP^\mu \quad [P^\mu, K^\nu] = 2i\eta^{\mu\nu}Z \quad [K^\mu, K^\nu] = 4iM^{\mu\nu} \\ \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu = \{S_\alpha, \bar{S}_{\dot{\alpha}}\} \\ [M^{\mu\nu}, Q_\alpha] &= -\frac{1}{2}\sigma_\alpha^{\mu\nu\beta}Q_\beta \quad [M^{\mu\nu}, \bar{Q}_{\dot{\alpha}}] = -\frac{1}{2}\bar{\sigma}_{\dot{\alpha}\dot{\beta}}^{\mu\nu}\bar{Q}^{\dot{\beta}} \\ [M^{\mu\nu}, S_\alpha] &= -\frac{1}{2}\sigma_\alpha^{\mu\nu\beta}S_\beta \quad [M^{\mu\nu}, \bar{S}_{\dot{\alpha}}] = -\frac{1}{2}\bar{\sigma}_{\dot{\alpha}\dot{\beta}}^{\mu\nu}\bar{S}^{\dot{\beta}} \\ \{Q_\alpha, S_\beta\} &= -2\epsilon_{\alpha\beta}Z \quad \{\bar{Q}_{\dot{\alpha}}, \bar{S}_{\dot{\beta}}\} = -2\epsilon_{\dot{\alpha}\dot{\beta}}Z \\ [K^\mu, Q_\alpha] &= -i\sigma_{\alpha\dot{\alpha}}^\mu \bar{S}^{\dot{\alpha}} \quad [K^\mu, \bar{Q}^{\dot{\alpha}}] = +i\bar{\sigma}^{\mu\dot{\alpha}\alpha}S_\alpha \\ [K^\mu S_\alpha] &= +i\sigma_{\alpha\dot{\alpha}}^\mu \bar{Q}^{\dot{\alpha}} \quad [K^\mu, \bar{S}_{\dot{\alpha}}] = -i\bar{\sigma}^{\mu\dot{\alpha}\alpha}Q_\alpha \\ [R, Q_\alpha] &= +Q_\alpha \quad [R, \bar{Q}_{\dot{\alpha}}] = -\bar{Q}_{\dot{\alpha}} \\ [R, S_\alpha] &= -S_\alpha \quad [R, \bar{S}_{\dot{\alpha}}] = +\bar{S}_{\dot{\alpha}} \end{aligned} \quad (A.4)$$

References

- [1] P. Horava and E. Witten, Nucl. Phys. **B460**, 506 (1996) [arXiv:hep-th/9510209]; N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B429**, 263 (1998) [arXiv:hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B436**, 257 (1998) [arXiv:hep-ph/9804398]; L. Randall and R. Sundrum, Phys. Rev. Lett.

83, 3370 (1999) [arXiv:hep-ph/9905221]; Phys. Rev. Lett. **83**, 4690 (1999) [arXiv:hep-th/9906064].

[2] M. Arai, M. Naganuma, M. Nitta, and N. Sakai, Nucl. Phys. **B652**, 35 (2003) [arXiv:hep-th/0211103]; “BPS Wall in N=2 SUSY Nonlinear Sigma Model with Eguchi-Hanson Manifold” in Garden of Quanta - In honor of Hiroshi Ezawa, Eds. by J. Arafune et al. (World Scientific Publishing Co. Pte. Ltd. Singapore, 2003) pp. 299-325 [arXiv:hep-th/0302028]; M. Arai, E. Ivanov and J. Niederle, Nucl. Phys. **B680**, 23 (2004) [arXiv:hep-th/0312037].

[3] Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, to appear in Phys. Rev. Lett., [arXiv:hep-th/0404198]; to appear in Phys. Rev. **D**, [arXiv:hep-th/0405194]; arXiv:hep-th/0405129.

[4] M. Naganuma, M. Nitta and N. Sakai, Grav. Cosmol. **8**, 129 (2002) [arXiv:hep-th/0108133]; R. Portugues and P. K. Townsend, JHEP **0204**, 039 (2002) [arXiv:hep-th/0203181].

[5] M. Eto, M. Nitta and N. Sakai, to appear in Nucl. Phys. **B**, [arXiv:hep-th/0405161].

[6] B. Chibisov and M. A. Shifman, Phys. Rev. D **56**, 7990 (1997) [Erratum-ibid. D **58**, 109901 (1998)] [arXiv:hep-th/9706141].

[7] D. V. Volkov and V. P. Akulov, JETP Lett. **16**, 438 (1972).

[8] S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2239 (1969); C. G. Callan, S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2247 (1969); D. V. Volkov, Sov. J. Particles and Nuclei **4**, 3 (1973); V. I. Ogievetsky, Proceedings of the X-th Winter School of Theoretical Physics in Karpacz, vol. 1, p. 227 (Wroclaw, 1974).

[9] E. Ivanov and S. Krivonos, Phys. Lett. B **453** (1999) 237 [arXiv:hep-th/9901003]; S. Bellucci, E. Ivanov and S. Krivonos, Phys. Lett. B **482**, 233 (2000) [arXiv:hep-th/0003273]; Nucl. Phys. Proc. Suppl.

102, 26 (2001) [arXiv:hep-th/0103136]; J. Hughes, J. Liu and J. Polchinski, Phys. Lett. B **180**, 370 (1986); J. Hughes and J. Polchinski, Nucl. Phys. B **278**, 147 (1986); J. Bagger and A. Galperin, Phys. Lett. B **336**, 25 (1994) [arXiv:hep-th/9406217].

[10] J. Bagger and A. Galperin, Phys. Rev. D **55**, 1091 (1997) [arXiv:hep-th/9608177]; M. Rocek and A. A. Tseytlin, Phys. Rev. D **59**, 106001 (1999) [arXiv:hep-th/9811232]; E. Ivanov, published in “Karpacz 1999, New symmetries and integrable models”, pp.206-217 [arXiv:hep-th/0002204].

[11] T. E. Clark, M. Nitta and T. ter Veldhuis, Phys. Rev. D **67**, 085026 (2003) [arXiv:hep-th/0208184]; Phys. Rev. D **69**, 047701 (2004) [arXiv:hep-th/0209142]; arXiv:hep-th/0409030.

[12] M.B. Green and J.H. Schwarz, Phys. Lett. **136B**, 367 (1984); Nucl. Phys. **B243** 285 (1984).

[13] A. Sen, JHEP **9910**, 008 (1999) [arXiv:hep-th/9909062].

[14] T. E. Clark, M. Nitta and T. ter Veldhuis, to appear in Phys. Rev. D, [arXiv:hep-th/0401163].

[15] M. Eto, N. Maru and N. Sakai, to appear in Nucl. Phys. B, [arXiv:hep-th/0404114].

[16] M. Arai, M. Nitta and N. Sakai, arXiv:hep-th/0307274; to appear in the Proceedings of the 3rd International Symposium on Quantum Theory and Symmetries (QTS3), September 10-14, 2003, [arXiv:hep-th/0401084]; to appear in the Proceedings of the International Conference on “Symmetry Methods in Physics (SYM-PHYS10)” held at Yerevan, Armenia, 13-19 Aug. 2003, [arXiv:hep-th/0401102]; to appear in the Proceedings of SUSY 2003 held at the University of Arizona, Tucson, AZ, June 5-10, 2003, [arXiv:hep-th/0402065].

[17] E. A. Ivanov and V. I. Ogievetsky, Teor. Mat. Fiz. **25**, 164 (1975).

- [18] K. Hayashi, Phys. Lett. B **44**, 497 (1973); M. Kalb and P. Ramond, Phys. Rev. D **9**, 2273 (1974); E. Cremmer and J. Scherk, Nucl. Phys. B **72**, 117 (1974); V. I. Ogievetsky and I. V. Polubarinov, Sov. J. Nucl. Phys. **4**, 156 (1967) [Yad. Fiz. **4**, 216 (1966)]; D. Z. Freedman and P. K. Townsend, Nucl. Phys. B **177**, 282 (1981); K. Seo, M. Okawa and A. Sugamoto, Phys. Rev. D **19**, 3744 (1979); T. E. Clark, C. H. Lee and S. T. Love, Nucl. Phys. B **308**, 379 (1988); Mod. Phys. Lett. A **4**, 1343 (1989); K. Furuta, T. Inami, H. Nakajima and M. Nitta, Prog. Theor. Phys. **106**, 851 (2001) [arXiv:hep-th/0106183].
- [19] J. Bagger and A. Galperin, Phys. Lett. B **412**, 296 (1997) [arXiv:hep-th/9707061].
- [20] A. A. Tseytlin, Nucl. Phys. B **469**, 51 (1996) [arXiv:hep-th/9602064].
- [21] A. Ritz, M. Shifman and A. Vainshtein, Phys. Rev. D **66**, 065015 (2002) [arXiv:hep-th/0205083]; arXiv:hep-th/0405175.
- [22] V. Markov, A. Marshakov and A. Yung, arXiv:hep-th/0408235.